

Automated Reasoning: Exercise 1

This work forms part of the coursework assessment. The deadline is November 19, 2009, 9am. Please submit your work to the Student Support Office, Room LF21.

Attempt all of the following questions.

Question 1 The following formula has its parentheses removed according to the priorities of connectives. Restore the parentheses (2 marks)

$$\neg p_1 \rightarrow \neg \neg p_2 \leftrightarrow p_3 \wedge p_4.$$

Question 2 The pigeonhole problem is the following problem (or rather a family of problems for each value of the parameters k, m described below). There are k pigeons and m holes. Is it possible to assign pigeons to holes so that every pigeon occupies some hole and no hole contains more than one pigeon? Formalize this problem, for each pair k, m , as a propositional satisfiability problem. *Hint:* use the boolean variables $\{p_{i,j} | i = 1 \dots k, j = 1 \dots m\}$, where $p_{i,j}$ denotes that the pigeon i occupies the hole j (5 marks).

Question 3 Show satisfiability of the following set of clauses using ordered resolution with the order $q \succ p \succ r$ (4 marks):

$$p \vee q \vee r, \neg q \vee r, \neg p \vee \neg q \vee r, q \vee \neg p.$$

Question 4 Show unsatisfiability of the following set of clauses using binary resolution (4 marks):

$$\neg q \vee r, \neg p \vee r, p \vee q, \neg p \vee \neg r, \neg q \vee \neg r.$$