Outline

Resolution

Inference Systems Soundness and Completeness Literal Selection and Orderings Inference Processes Redundancy Elimination

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Binary Resolution Inference System

The binary resolution inference system, denoted by \mathbb{BR} , consists of two inference rules:

Binary resolution, denoted by BR

$$\frac{p \lor C_1 \quad \neg p \lor C_2}{C_1 \lor C_2}$$
 (BR).

Factoring, denoted by Fact:

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inference has the form

$$\frac{F_1 \quad \dots \quad F_n}{G} \; ,$$

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- The formula G is called the conclusion of the inference;
- The formulas F_1, \ldots, F_n are called its premises.
- ► An inference rule *R* is a set of inferences.
- Every inference $I \in R$ is called an instance of R.
- ► An Inference system I is a set of inference rules.
- Axiom: inference rule with no premises.

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Derivation, Proof

- Derivation in an inference system I: a tree built from inferences in I.
- If the root of this derivation is *E*, then we say it is a derivation of *E*.
- Proof of E: a finite derivation whose leaves are axioms.
- ► Derivation of *E* from *E*₁,..., *E_m*: a finite derivation of *E* whose every leaf is either an axiom or one of the expressions *E*₁,..., *E_m*.

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Soundness

- An inference is sound if the conclusion of this inference is a logical consequence of its premises.
- An inference rule is sound if every inference of this rule is sound.

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Theorem BR *is sound.*

Consequence of Soundness

Theorem Let *S* be a set of clauses. If \Box can be derived from *S* in $\mathbb{B}\mathbb{R}$, then *S* is unsatisfiable.

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Example

Consider the following set of clauses

$$\{\neg p \lor \neg q, \ \neg p \lor q, \ p \lor \neg q, \ p \lor q\}.$$

The following derivation derives the empty clause from this set:

$$\frac{p \lor q \quad p \lor \neg q}{\frac{p \lor p}{p} \text{ (Fact)}} (BR) \quad \frac{\neg p \lor q \quad \neg p \lor \neg q}{\frac{\neg p \lor \neg p}{p} \text{ (Fact)}} (BR)$$

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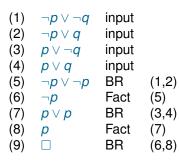
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Writing derivations in the linear form

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Completeness

 \mathbb{BR} is complete, that is, if a set of clauses is unsatisfiable, then one can derive an empty clause from this set.

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The binary resolution inference system has too many inferences. There are restrictions on resolution that allow for fewer inferences but preserve completeness.

To define these systems we need a new notion.

A literal selection function selects one or more literals in every non-empty clause.

We will sometimes denote selected literals by underlining them, e.g.,

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Unrestricted binary resolution and binary resolution with selection

Consider the selection function that selects all literals in a clause. Then the binary resolution rule:

$$\frac{p \lor C_1 \quad \neg p \lor C_2}{C_1 \lor C_2}$$
 (BR).

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becomes a special case of binary resolution with selection.

Literal Orderings

Consider any total ordering \succ on propositional variables. We want to extend it to literals.

Let $L_1 = (\neg)A_1$ and $L_2 = (\neg)A_2$ be literals. We let $L_1 \succ_{lit} L_2$ if and only if one of the following conditions holds:

- 1. $A_1 \succ A_2$; or
- 2. $A_1 = A_2$, L_1 is negative and L_2 is positive.

In other words, we compare literals by first comparing the atoms of these literals and if the atoms are equal define the negative literal to be greater.

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Fix an ordering \succ on the set of propositional variables and let \succ_{lit} be corresponding literal ordering. Consider the selection function σ that selects all maximal w.r.t. \succ_{lit} literals.

Theorem

 \mathbb{BR}_{σ} is complete, that is, for every unsatisfiable set of clauses *S* one can derive the empty clause from clauses in *S* using inferences in \mathbb{BR}_{σ} .

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Ordered resolution: example

Assume $q \succ p$.			
(1)	$ eg p \lor eg q$	input	
(2)	$\neg p \lor \overline{q}$	input	
(3)	$p \lor \neg \overline{q}$	input	
(4)	$p \lor \overline{q}$	input	
(5)	$\neg p \overline{\vee} \neg p$	BR	(1,2)
(6)	$\overline{p \lor p}$	BR	(3,4)
(7)	\overline{p} –	Fact	(6)
(8)	$\neg p$	BR	(6,7)
(9)		BR	(6,8)

Note: fewer inferences are enabled compared to unrestricted binary resolution.

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Ordered resolution: example

Assume $q \succ p$. (1) input $\neg p \lor \neg q$ (2) $\neg p \lor q$ input (3) $p \vee \neg q$ input (4) $p \lor q$ input (5) <u>¬</u>*p* √ ¬*p* BR (1,2) $\begin{array}{ccc} (6) & \underline{p} \lor \underline{p} \\ (7) & \underline{p} \end{array}$ BR (3,4)Fact (6)(8) $\neg p$ BR (6,7)(9) BR (6,8)

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Inference Process

Inference process: sequence of sets of clauses S_0, S_1, \ldots , denoted by

 $S_0 \Rightarrow S_1 \Rightarrow S_2 \Rightarrow \dots$

 $(S_i \Rightarrow S_{i+1})$ is a step of this process.

We say that this step is an I-step if

1. there exists an inference

$$\frac{C_1 \quad \dots \quad C_n}{C}$$

in I such that $\{C_1, \ldots, C_n\} \subseteq S_i;$ 2. $S_{i+1} = S_i \cup \{C\}.$

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An $\mathbb{I}\text{-inference process}$ is an inference process whose every step is an $\mathbb{I}\text{-step}.$

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Property

Lemma Let $S_0 \Rightarrow S_1 \Rightarrow S_2 \Rightarrow \dots$ be an \mathbb{I} -inference process and a clause *C* belongs to some S_i . Then S_i is derivable in \mathbb{I} from S_0 .

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Can we prove the inverse?

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Can we prove the inverse?

Limit and Fairness

The limit of an inference process $S_0 \Rightarrow S_1 \Rightarrow S_2 \Rightarrow \dots$ is the set of clauses $\bigcup_i S_i$.

Let $S_0 \Rightarrow S_1 \Rightarrow S_2 \Rightarrow \dots$ be an inference process with the limit S_{∞} . The process is called fair if for every I-inference

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if $\{C_1, \ldots, C_n\} \subseteq S_{\infty}$, then there exists *i* such that $C \in S_i$.

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Completeness, reformulated

Theorem Let ${\rm I\hspace{-.1em}I}$ be an inference system. The following conditions are equivalent.

- 1. I is complete.
- 2. For every unsatisfiable set of clauses S_0 and any fair I-inference process with the initial set S_0 , the limit of this inference process contains \Box .

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Let \mathbb{I} be an inference system and S be a set of clauses. S is called saturated with respect to \mathbb{I} , or simply \mathbb{I} -saturated, if for every inference of \mathbb{I} with premises in S, the conclusion of this inference also belongs to S.

The closure of *S* with respect to \mathbb{I} , or simply \mathbb{I} -closure, is the smallest set *S*' containing *S* and saturated with respect to \mathbb{I} .

Completeness of Ordered Resolution

Theorem (Completeness)

Take any well-founded ordering \succ and consider the selection function σ that selects all maximal w.r.t. \succ_{lit} literals. Let S_0 be a set of clauses and $S_0 \Rightarrow S_1 \Rightarrow S_2 \Rightarrow \ldots$ be a fair \mathbb{BR}_{σ} -inference process. Then S_0 is unsatisfiable if and only if $\Box \in S_i$ for some *i*.

Lemma The limit S_{ω} is saturated.

Lemma The limit S_{ω} is logically equivalent to the initial set S_0 .

Lemma A saturated set S of clauses is unsatisfiable if and only if $\Box \in S$.

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Corollaries

Completeness of Binary Resolution. Binary resolution is complete. **Compactness.** Let S be a countably infinite set of clauses. Then S is unsatisfiable if and only if it contains a finite unsatisfiable subset. **Note.** The assumption of being countably infinite can be dropped.

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Problem: search space grows too fast

Idea: remove some clauses from the search space. We will consider later how clauses can be removed without compromising completeness.

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Inference Process with Deletion

Let I be an inference system. Consider an inference process with two kinds of step $S_i \Rightarrow S_{i+1}$:

- 1. I-inference;
- 2. deletion of a clause in S_i , that is

$$S_{i+1}=S_i-\{C\},$$

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where $C \in S_i$.

Fairness: Persistent Clauses and Limit

Consider an inference process

 $S_0 \Rightarrow S_1 \Rightarrow S_2 \Rightarrow \dots$

A clause C is called persistent if

 $\exists i \forall j \geq i (C \in S_j).$

The limit S_{ω} of the inference process is the set of all persistent clauses:

$$S_\omega = igcup_{i=0,1,...}igcup_{j\geq i}S_j.$$

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The process is called I-fair if every inference with persistent premises in S_{ω} has been applied, that is, if

$$\frac{C_1 \quad \dots \quad C_r}{C}$$

is an inference in \mathbb{I} and $\{C_1, \ldots, C_n\} \subseteq S_{\omega}$, then $C \in S_i$ for some *i*.

Deletion rules

Tautology: a clause of the form $p \lor \neg p \lor C$. Tautology deletion: deletion of tautologies from the search space.

Finite multiset: like a set but elements may occur more than once. Example: $\{1, 2, 2, 5, 5, 5\}$. A clause can be considered as a multiset of its literals.

A clause C_1 is said to subsume any clause $C_1 \vee C_2$, where C_2 is non-empty. In other words, C_1 subsumes C_2 if and only if C_1 is a submultiset of C_2 .

Subsumption deletion: deletion of subsumed clauses from the search space.

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Completeness with deletion rules

Subsumption and tautology deletion does not compromise completeness of binary and ordered resolution. That is, for every fair inference process with subsumption a tautology deletion, if the initial set of clauses is unsatisfiable, then the limit of the process contains the empty clause.

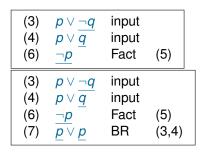
Example: inference process with deletion

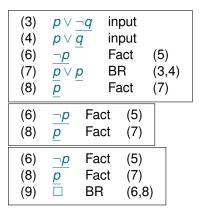
(1) (2) (3) (4)	$ \begin{array}{c} \neg p \lor \neg q \\ \neg p \lor \overline{q} \\ p \lor \neg \overline{q} \\ p \lor \overline{q} \end{array} $	input input input input	
(1) (2) (3) (4) (5)	$ \begin{array}{c} \neg p \lor \neg q \\ \neg p \lor \overline{q} \\ p \lor \neg q \\ p \lor \overline{q} \\ \neg p \lor \overline{q} \\ \neg p \lor \neg p \end{array} $	input input input input BR	(1,2)

(1) (2) (3) (4) (5) (6)	$ \begin{array}{c} \neg p \lor \neg q \\ \neg p \lor q \\ p \lor \neg q \\ p \lor q \\ \neg p \lor q \\ \neg p \\ \neg p \\ \end{array} $	input input input BR Fact	(1,2 (5)	2)
(3) (4) (6)	$ \begin{array}{c} p \lor \underline{\neg q} \\ p \lor \underline{q} \\ \underline{\neg p} \end{array} $	input input Fact	(5)	

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Example: inference process with deletion





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