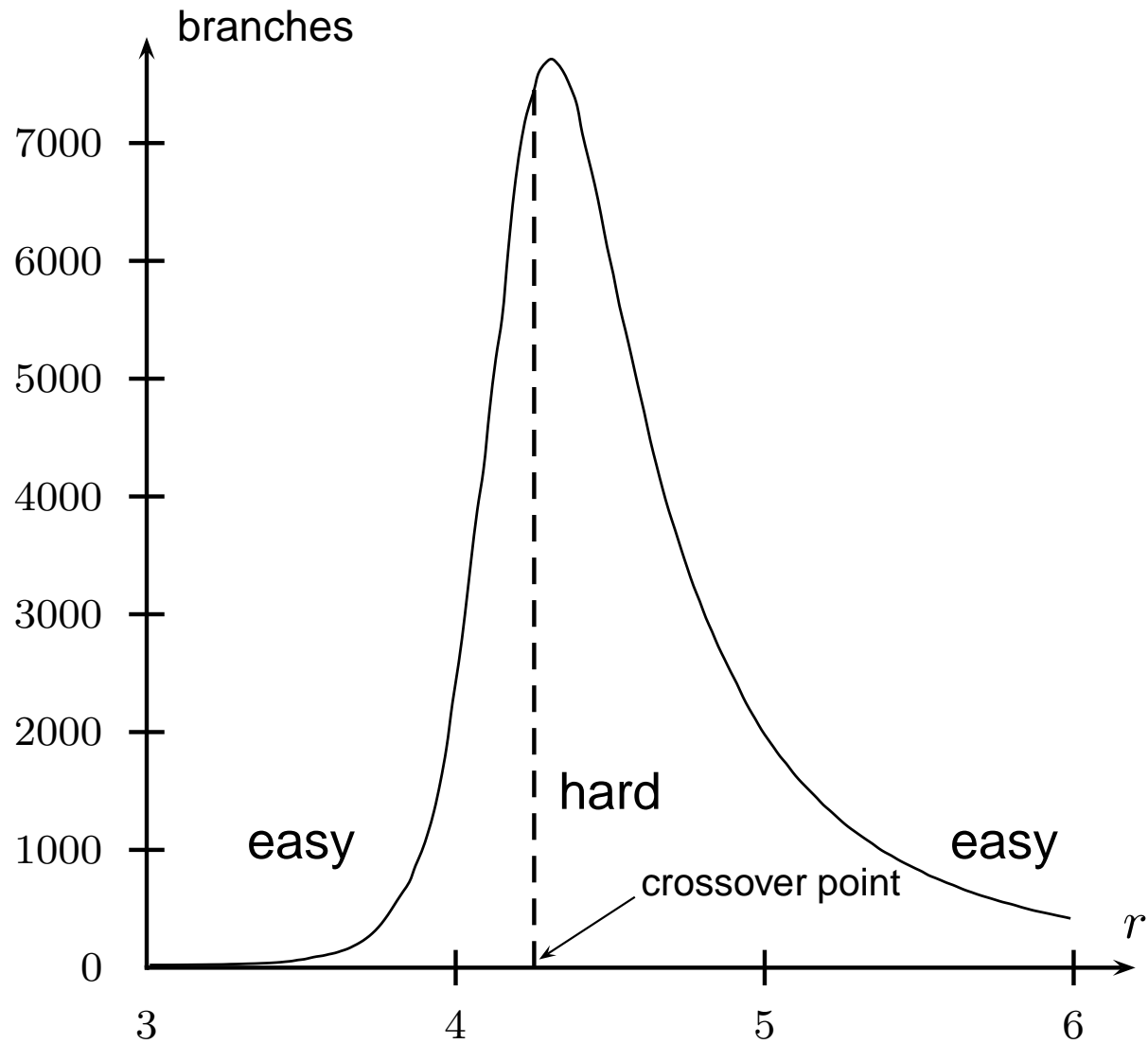


Easy-Hard-Easy Pattern



Satisfiability Algorithm that Cannot Establish Unsatisfiability

procedure *CHAOS*(*S*)

input: set of clauses *S*

output: interpretation *I* such that $I \models S$ or *don't know*

parameters: positive integer *MAX-TRIES*

begin

repeat *MAX-TRIES* times

I := random interpretation

if $I \models S$ then return *I*

return *don't know*

end

Satisfiability has **short witnesses**: interpretations.

Randomised Algorithms for SAT

- ▶ Choose a **random interpretation**.
- ▶ If this interpretation is not a model, repeatedly choose a variable and **change its value in the interpretation** (**flip** the variable).

The flipped variables are chosen using heuristics or randomly, or both.

Flipping a Variable

$$\mathit{flip}(I, p)(q) = \begin{cases} I(q), & \text{if } p \neq q; \\ 1, & \text{if } p = q \text{ and } I(p) = 0; \\ 0, & \text{if } p = q \text{ and } I(p) = 1. \end{cases}$$

In other words, the interpretation $\mathit{flip}(I, p)$ is obtained from I by changing its value on p .

GSAT

procedure $GSAT(S)$

input: set of clauses S

output: interpretation I such that $I \models S$ or *don't know*

parameters: integers $MAX-TRIES$, $MAX-FLIPS$

begin

repeat $MAX-TRIES$ times

$I :=$ random interpretation

if $I \models S$ then return I

repeat $MAX-FLIPS$ times

$p :=$ an atom such that $flip(I, p)$ satisfies

the maximal number of clauses in S

$I = flip(I, p)$

if $I \models S$ then return I

return *don't know*

end

GSAT example

$p_1 \vee \neg p_2 \vee p_3, \neg p_2 \vee \neg p_3, \neg p_1 \vee \neg p_3, \neg p_1 \vee p_2, p_1 \vee p_2.$

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped atom
	p_1	p_2	p_3	p_1	p_2	p_3		
1	0	0	1	4				

GSAT example

$p_1 \vee \neg p_2 \vee p_3, \quad \neg p_2 \vee \neg p_3, \quad \neg p_1 \vee \neg p_3, \quad \neg p_1 \vee p_2, \quad p_1 \vee p_2.$

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped atom	
	p_1	p_2	p_3	p_1	p_2	p_3			
1	0	0	1	4	3	4	4	p_2, p_3	p_2

GSAT example

$p_1 \vee \neg p_2 \vee p_3, \quad \neg p_2 \vee \neg p_3, \quad \neg p_1 \vee \neg p_3, \quad \neg p_1 \vee p_2, \quad p_1 \vee p_2.$

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped atom	
	p_1	p_2	p_3	p_1	p_2	p_3			
1	0	0	1	4	3	4	4	p_2, p_3	p_2
2	0	1	1	4					

GSAT example

$p_1 \vee \neg p_2 \vee p_3, \quad \neg p_2 \vee \neg p_3, \quad \neg p_1 \vee \neg p_3, \quad \neg p_1 \vee p_2, \quad p_1 \vee p_2.$

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped atom
	p_1	p_2	p_3		p_1	p_2	p_3		
1	0	0	1	4	3	4	4	p_2, p_3	p_2
2	0	1	1	4	3	4	4	p_2, p_3	p_3

GSAT example

$p_1 \vee \neg p_2 \vee p_3, \quad \neg p_2 \vee \neg p_3, \quad \neg p_1 \vee \neg p_3, \quad \neg p_1 \vee p_2, \quad p_1 \vee p_2.$

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped atom
	p_1	p_2	p_3		p_1	p_2	p_3		
1	0	0	1	4	3	4	4	p_2, p_3	p_2
2	0	1	1	4	3	4	4	p_2, p_3	p_3
3	0	1	0	4					

GSAT example

$p_1 \vee \neg p_2 \vee p_3, \quad \neg p_2 \vee \neg p_3, \quad \neg p_1 \vee \neg p_3, \quad \neg p_1 \vee p_2, \quad p_1 \vee p_2.$

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped atom
	p_1	p_2	p_3		p_1	p_2	p_3		
1	0	0	1	4	3	4	4	p_2, p_3	p_2
2	0	1	1	4	3	4	4	p_2, p_3	p_3
3	0	1	0	4	5	4	4	p_1	p_1

GSAT example

$p_1 \vee \neg p_2 \vee p_3, \quad \neg p_2 \vee \neg p_3, \quad \neg p_1 \vee \neg p_3, \quad \neg p_1 \vee p_2, \quad p_1 \vee p_2.$

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped atom	
	p_1	p_2	p_3	p_1	p_2	p_3			
1	0	0	1	4	3	4	4	p_2, p_3	p_2
2	0	1	1	4	3	4	4	p_2, p_3	p_3
3	0	1	0	4	5	4	4	p_1	p_1
	1	1	0	5					

GSAT with random walks

procedure *GSATwithWalks*(S)

input: set of clauses S

output: interpretation I such that $I \models S$ or *don't know*

parameters: integers *MAX-TRIES*, *MAX-FLIPS*

real number $0 \leq \pi \leq 1$ (probability of a sideways move),

begin

repeat *MAX-TRIES* times

$I :=$ random interpretation ; if $I \models S$ then return I

repeat *MAX-FLIPS* times

with probability π

$p :=$ an atom such that $flip(I, p)$ satisfies the maximal number of clauses in S

with probability $1 - \pi$

randomly select p among atoms occurring in clauses false in I

$I = flip(I, p)$; if $I \models S$ then return I

return *don't know*

end

WSAT

procedure $WSAT(S)$

input: set of clauses S

output: interpretation I such that $I \models S$ or *don't know*

parameters: integers $MAX-TRIES$, $MAX-FLIPS$

begin

repeat $MAX-TRIES$ times

$I :=$ random interpretation

if $I \models S$ then return I

repeat $MAX-FLIPS$ times

randomly select a clause $C \in S$ such that $I \not\models C$

randomly select an atom p in C

$I = flip(I, p)$

if $I \models S$ then return I

return *don't know*

end

WSAT example

$p_1 \vee \neg p_2 \vee p_3, \neg p_2 \vee \neg p_3, \neg p_1 \vee \neg p_3, \neg p_1 \vee p_2, p_1 \vee p_2.$

flip	interpretation			unsatisfied	candidates	flipped
no.	p_1	p_2	p_3	clauses	for flipping	atom
1	0	0	1	$p_1 \vee p_2$		

WSAT example

$p_1 \vee \neg p_2 \vee p_3, \neg p_2 \vee \neg p_3, \neg p_1 \vee \neg p_3, \neg p_1 \vee p_2, p_1 \vee p_2.$

flip	interpretation			unsatisfied	candidates	flipped
no.	p_1	p_2	p_3	clauses	for flipping	atom
1	0	0	1	$p_1 \vee p_2$	p_1, p_2	p_1

WSAT example

$p_1 \vee \neg p_2 \vee p_3, \neg p_2 \vee \neg p_3, \neg p_1 \vee \neg p_3, \neg p_1 \vee p_2, p_1 \vee p_2.$

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped atom
	p_1	p_2	p_3			
1	0	0	1	$p_1 \vee p_2$	p_1, p_2	p_1
2	1	0	1	$\neg p_1 \vee \neg p_3$ $\neg p_1 \vee p_2$		

WSAT example

$p_1 \vee \neg p_2 \vee p_3, \neg p_2 \vee \neg p_3, \neg p_1 \vee \neg p_3, \neg p_1 \vee p_2, p_1 \vee p_2.$

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped atom
	p_1	p_2	p_3			
1	0	0	1	$p_1 \vee p_2$	p_1, p_2	p_1
2	1	0	1	$\neg p_1 \vee \neg p_3$ $\neg p_1 \vee p_2$	p_1, p_2, p_3	p_2

WSAT example

$p_1 \vee \neg p_2 \vee p_3, \neg p_2 \vee \neg p_3, \neg p_1 \vee \neg p_3, \neg p_1 \vee p_2, p_1 \vee p_2.$

flip no.	interpretation			unsatisfied	candidates	flipped
	p_1	p_2	p_3	clauses	for flipping	atom
1	0	0	1	$p_1 \vee p_2$	p_1, p_2	p_1
2	1	0	1	$\neg p_1 \vee \neg p_3$ $\neg p_1 \vee p_2$	p_1, p_2, p_3	p_2
3	1	1	1	$\neg p_2 \vee \neg p_3$ $\neg p_1 \vee \neg p_3$		

WSAT example

$p_1 \vee \neg p_2 \vee p_3, \neg p_2 \vee \neg p_3, \neg p_1 \vee \neg p_3, \neg p_1 \vee p_2, p_1 \vee p_2.$

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped atom
	p_1	p_2	p_3			
1	0	0	1	$p_1 \vee p_2$	p_1, p_2	p_1
2	1	0	1	$\neg p_1 \vee \neg p_3$ $\neg p_1 \vee p_2$	p_1, p_2, p_3	p_2
3	1	1	1	$\neg p_2 \vee \neg p_3$ $\neg p_1 \vee \neg p_3$	p_1, p_2, p_3	p_3

WSAT example

$p_1 \vee \neg p_2 \vee p_3, \neg p_2 \vee \neg p_3, \neg p_1 \vee \neg p_3, \neg p_1 \vee p_2, p_1 \vee p_2.$

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped atom
	p_1	p_2	p_3			
1	0	0	1	$p_1 \vee p_2$	p_1, p_2	p_1
2	1	0	1	$\neg p_1 \vee \neg p_3$ $\neg p_1 \vee p_2$	p_1, p_2, p_3	p_2
3	1	1	1	$\neg p_2 \vee \neg p_3$ $\neg p_1 \vee \neg p_3$	p_1, p_2, p_3	p_3
	1	1	0			