Outline

Transition Systems
- State-Changing Systems
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- Symbolic Representation of Transition Systems
Our main interest from now on is modelling state-changing systems.

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2. Using a **logic to specify and verify properties** of the system.
Vending Machine Example

Consider an example state-changing system: a vending machine which dispenses drinks in a university department.

- The machine has several components, including at least the following: a storage space for storing and preparing drinks, a box for dispensing drinks and a coin slot for putting coins in.
- When the machine is operating, it goes through several states depending on the behavior of the current customer.
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- Each action undertaken by the customer or by the machine itself may change the state of the machine. For example, when the customer inserts a coin in the coin slot, the amount of money stored in the slot changes.
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- Each action undertaken by the customer or by the machine itself may change the state of the machine. For example, when the customer inserts a coin in the coin slot, the amount of money stored in the slot changes.
- Actions which may change the state of the system are called transitions.
Modeling State-Changing Systems

To build a formal model of a particular state-changing system, we should define

1. What are the state variables.
2. What are the possible values of the state variables.
3. What are the transitions and how they change the values of the state variables.
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A state can be identified with the set of pairs \((\text{variable}, \text{value})\), or with a function from variables to values.
Transition Systems

A transition system is a tuple $\mathcal{S} = (S, \text{In}, T, \mathcal{X}, \text{dom}, L)$, where

1. $S$ is a finite non-empty set, called the set of states of $\mathcal{S}$.
2. $\text{In} \subseteq S$ is a non-empty set of states, called the set of initial states of $M$. 
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$\mathcal{X}$, $\text{dom}$ and $L$ will be explained later.
State Transition Graph

State Transition Graph of a transition system $S$:

- The **nodes** are the states of $S$.
- The **arcs** are elements of the transition relation: there is an arc from a state $s$ to a state $s'$ if and only if $(s, s') \in T$.

We denote the initial state(s) using double lines.
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5. $\text{dom}$ is a mapping from $X$ such that for every state variable $v \in X$, $\text{dom}(v)$ is a non-empty set, called the domain for $v$. 

The transition system is said to be finite-state if for every state variable $v$, the domain $\text{dom}(v)$ for this variable is finite.

We will only study finite-state transition systems.
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6. $L$ is a function mapping states in $S$ into interpretations, called the labeling function of $S$. It will be explained later.
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That is, for a transition system $S = (S, \text{In}, T, \mathcal{X}, \text{dom}, L)$, the set of variables $\mathcal{X}$ and the mapping $\text{dom}$ defines an instance of propositional logic of finite domains.
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Denote the set of all interpretations for this instance of PLFD by $\mathbb{I}$. Then the labelling function $L$ is a mapping $L : S \rightarrow \mathbb{I}$, that is, it maps every state to an interpretation.
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Denote the set of all interpretations for this instance of PLFD by $\mathbb{I}$. Then the labelling function $L$ is a mapping $L : S \rightarrow \mathbb{I}$, that is, it maps every state to an interpretation.

This means that

1. for every variable $v \in \mathcal{X}$ and every state $s \in S$, we have $L(s)(v) \in \text{dom}(v)$;

2. for every formula $A$ of this instance of PLFD and every state $s \in S$, either $L(s) \models A$ or $L(s) \not\models A$. 
State Transition Graph of a transition system $S$:

- The nodes are the states of $S$.
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Assume two boolean-valued variables $x, y$:

- $x = 1, y = 0$
- $x = 1, y = 1$
- $x = 0, y = 1$
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State Transition Graph of a transition system $\mathcal{S}$:

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Assume two boolean-valued variables $x, y$.

We denote the initial state(s) using double lines.
States as Interpretations

Essentially, in each state each variable has a value.

- If $L(s)(x) = v$ then we say that $x$ has the value $v$ in $s$ and write $s(x) = v$.
- If $L(s) \models A$ then we say that $s$ satisfies $A$ or $A$ is true in $s$ and write $s \models A$.

In both cases, we identify $s$ with $L(s)$. 
States as Interpretations

\[ x = 1 \]
\[ y = 0 \]

\[ x = 1 \]
\[ y = 1 \]

\[ x = 0 \]
\[ y = 0 \]

\[ x = 0 \]
\[ y = 1 \]
States as Interpretations

\[ s_1 \models x \]

\[
\begin{align*}
  s_1 & \models x = 1 \quad y = 0 \\
  s_2 & \models x = 1 \quad y = 1 \\
  s_3 & \models x = 0 \quad y = 0 \\
  s_4 & \models x = 0 \quad y = 1
\end{align*}
\]
States as Interpretations

- $s_1$: $x = 1, y = 0$
- $s_2$: $x = 1, y = 1$
- $s_3$: $x = 0, y = 0$
- $s_4$: $x = 0, y = 1$

- $s_1 \models x$
- $s_2 \models x \land y$
States as Interpretations

- $s_1$ \models x
- $s_2$ \models x \land y
- $s_3$ \models x \leftrightarrow y
Transitions

When we model systems, we will usually represent the transition relation as a union of so-called transitions.

- A transition $t$ is any set of pairs of states.
- A transition $t$ is applicable to a state $s$ if there exists a state $s'$ such that $(s, s') \in t$.
- A transition $t$ is deterministic if for every state $s$ there exists at most one state $s'$ such that $(s, s') \in t$. 
Vending Machine

1. The vending machine contains a drink storage, a coin slot, and a drink dispenser. The drink storage stores drinks of two kinds: beer and coffee. We are only interested in whether a particular kind of drink is currently being stored or not, but not interested in the amount of it.

2. The coin slot can accommodate up to three coins.

3. The drink dispenser can store at most one drink. If it contains a drink, this drink should be removed before the next one can be dispensed.

4. A can of beer costs two coins. A cup of coffee costs one coin.

5. There are two kinds of customers: students and professors. Students drink only beer, professors drink only coffee.

6. From time to time the drink storage can be recharged.
Vending Machine

1. The vending machine contains a **drink storage**, a **coin slot**, and a **drink dispenser**. The drink storage stores drinks of two kinds: **beer** and **coffee**. We are only interested in whether a particular kind of drink is currently being stored or not, but not interested in the amount of it.

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## Formalization: Variables and Domains

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<th>variable</th>
<th>domain</th>
<th>explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>st_coffee</td>
<td>{0, 1}</td>
<td>drink storage contains coffee</td>
</tr>
<tr>
<td>st_beer</td>
<td>{0, 1}</td>
<td>drink storage contains beer</td>
</tr>
<tr>
<td>disp</td>
<td>{none, beer, coffee}</td>
<td>content of drink dispenser</td>
</tr>
<tr>
<td>coins</td>
<td>{0, 1, 2, 3}</td>
<td>number of coins in the slot</td>
</tr>
<tr>
<td>customer</td>
<td>{none, student, prof}</td>
<td>customer</td>
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Transitions for the Vending Machine

1. *Recharge* which results in the drink storage having both beer and coffee.
2. *Customer_arrives*, after which a customer appears at the machine.
4. *Coin_insert*, when the customer inserts a coin in the machine.
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Symbolic Representation of Sets of States

Let $\mathcal{S} = (S, \text{In}, T, \chi, \text{dom}, L)$ be a finite-state transition system. Then every formula $F$ defines a set states:

$$\{s \mid s \models F\}.$$
Symbolic Representation of Sets of States

Let $\mathcal{S} = (S, \text{In}, T, \mathcal{X}, \text{dom}, L)$ be a finite-state transition system. Then every formula $F$ defines a set states:

$$\{s \mid s \models F\}.$$ 

We say that $F$ (symbolically) represent this set of states.
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- $s_1$: $x = 1$, $y = 0$
- $s_2$: $x = 1$, $y = 1$
- $s_3$: $x = 0$, $y = 0$
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$\leftrightarrow$ represents $\{s_2, s_3\}$

$\land$ represents $\{s_2\}$

$\neg$ represents $\{s_3, s_4\}$
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- $x \leftrightarrow y$ represents $\{s_2, s_3\}$
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- $x \land y$
Symbolic Representation of Sets of States

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Example

Let us represent the set of states in which the machine is ready to dispense a drink. In every such state, a drink should be available, the drink dispenser empty, and the coin slot contain enough coins.

\[(st_{coffee} \lor st_{beer}) \land disp = none \land (\text{coins} = 1 \lor \text{coins} = 2 \lor \text{coins} = 3)\].
Example

Let us represent the set of states in which the machine is ready to dispense a drink. In every such state, a drink should be available, the drink dispenser empty, and the coin slot contain enough coins.

This can be expressed by:

\[(\text{st\_coffee} \lor \text{st\_beer}) \land \text{disp} = \text{none} \land ((\text{coins} = 1 \land \text{st\_coffee}) \lor \text{coins} = 2 \lor \text{coins} = 3)\].
Symbolic Representation of Transitions

A transition is a relation on pairs of states. It brings the system to the current state and the next state. Formulas of PLFD can only express properties of a single state. How can we represent transitions using formulas?

In addition to the set of propositional variables $X = \{x_1, \ldots, x_n\}$, introduce a set of next state variables $X' = \{x'_1, \ldots, x'_n\}$.

Pairs of states as interpretations. For every variable $x \in X$ define $(s, s') (x) \overset{\text{def}}{=} s(x)$; $(s, s') (x') \overset{\text{def}}{=} s'(x)$.

Symbolic representation. Formula $F$ of variables $X \cup X'$ represents a transition $t$ if $t = \{ (s, s') | (s, s') | F \}$. 
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- Symbolic representation. Formula \( F \) of variables \( X \cup X' \) represents a transition \( t \) if \( t = \{ (s, s') \mid (s, s') \models F \} \).
Example

The transition *Recharge*:

\[
\text{customer} = \text{none} \land \text{st\_coffee}' \land \text{st\_beer}'.
\]
Example

The transition *Recharge*:

\[ \text{customer} = \text{none} \land \text{st\_coffee}' \land \text{st\_beer}'. \]

But this formula includes describes a very strange transition after which, for example

- coins may appear in and disappear from the slot;
- dfrinks may appear in and disappear from the dispenser.
- \ldots
Frame Problem

One has to express explicitly, maybe for a large number of state variables, that the values of these variables do not change after a transition. For example,

\[(\text{coins} = 0 \leftrightarrow \text{coins}^\prime = 0) \land \]
\[(\text{coins} = 1 \leftrightarrow \text{coins}^\prime = 1) \land \]
\[(\text{coins} = 2 \leftrightarrow \text{coins}^\prime = 2) \land \]
\[(\text{coins} = 3 \leftrightarrow \text{coins}^\prime = 3).\]

This frame problem arises in artificial intelligence, knowledge representation, and reasoning about actions.
End of Lecture 17

Slides for lecture 17 end here . . .
Abbreviations (we assume $\text{dom}(x) = \text{dom}(y)$):

\[
\begin{align*}
   x \neq v & \quad \text{def} \quad \neg (x = v) \\
   x = y & \quad \text{def} \quad \bigwedge_{v \in \text{dom}(x)} (x = v \iff y = v).
\end{align*}
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Let $S$ be a transition system and \{\(x_1, \ldots, x_n\)\} $\subseteq \mathcal{X}$ be a set of state variables of $\mathcal{L}(S)$. Define

\[
\text{only}(x_1, \ldots, x_n) \quad \overset{\text{def}}{=} \quad \bigwedge_{y \in \mathcal{X} \setminus \{x_1, \ldots, x_n\}} y = y'.
\]

This formula expresses that $x_1, \ldots, x_n$ are the only variables whose values can be changed by the transition.
When we represent a transition symbolically using a formula $F$ of variables $\mathcal{X} \cup \mathcal{X}'$, the formula $F$ is usually represented as the conjunction $F_1 \land F_2$ of two formulas:

1. $F_1$ expresses some conditions on the variables $\mathcal{X}$ which are necessary to execute the transition (precondition);
Preconditions and Postconditions

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1. $F_1$ expresses some conditions on the variables $\mathcal{X}$ which are necessary to execute the transition (precondition);
2. $F_2$ expresses some conditions relating variables in $\mathcal{X}$ to those in $\mathcal{X}'$, i.e., conditions which show how the values of the variables after the transition relate to their values before the transition (postcondition).
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Transitions: Symbolic Representation

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\text{st}_\text{beer} & \quad \{0, 1\} \\
\text{disp} & \quad \{\text{none, beer, coffee}\} \\
\text{coins} & \quad \{0, 1, 2, 3\} \\
\text{customer} & \quad \{\text{none, student, prof}\}
\end{align*}
\]

\[\text{Recharge} \quad \text{def} \quad \text{customer} = \text{none} \land \text{st}_\text{coffee}' \land \text{st}_\text{beer}' \land \text{only}(\text{st}_\text{coffee}, \text{st}_\text{beer}).\]
Transitions: Symbolic Representation

\[
\begin{align*}
st_{\text{coffee}} & \{0, 1\} \\
st_{\text{beer}} & \{0, 1\} \\
disp & \{\text{none, beer, coffee}\} \\
coins & \{0, 1, 2, 3\} \\
customer & \{\text{none, student, prof}\}
\end{align*}
\]

\[
Recharge \overset{\text{def}}{=} \text{customer} = \text{none} \land \\
\text{st}_{\text{coffee}}' \land \text{st}_{\text{beer}}' \land \\
\text{only}(\text{st}_{\text{coffee}}, \text{st}_{\text{beer}}).
\]
Transitions: Symbolic Representation

\begin{align*}
\text{st}_{\text{coffee}} & \quad \{0, 1\} \\
\text{st}_{\text{beer}} & \quad \{0, 1\} \\
\text{disp} & \quad \{\text{none, beer, coffee}\} \\
\text{coins} & \quad \{0, 1, 2, 3\} \\
\text{customer} & \quad \{\text{none, student, prof}\} \\
\text{Recharge} & \quad \text{def} = \text{customer} = \text{none} \land \\
& \quad \text{st}’_{\text{coffee}} \land \text{st}’_{\text{beer}} \land \\
& \quad \text{only} (\text{st}_{\text{coffee}}, \text{st}_{\text{beer}}). \\
\text{Customer}_{\text{arrives}} & \quad \text{def} = \text{customer} = \text{none} \land \text{customer} \neq \text{none} \land \\
& \quad \text{only} (\text{customer})
\end{align*}
Transitions: Symbolic Representation

- \( \text{st\_coffee} \): \( \{0, 1\} \)
- \( \text{st\_beer} \): \( \{0, 1\} \)
- \( \text{disp} \): \( \{\text{none, beer, coffee}\} \)
- \( \text{coins} \): \( \{0, 1, 2, 3\} \)
- \( \text{customer} \): \( \{\text{none, student, prof}\} \)

\[
\begin{align*}
\text{Recharge} \quad &\equiv \quad \text{customer} = \text{none} \land \\
&\quad \text{st\_coffee}' \land \text{st\_beer}' \land \\
&\quad \text{only}(\text{st\_coffee}, \text{st\_beer}).
\end{align*}
\]

\[
\begin{align*}
\text{Customer\_arrives} \quad &\equiv \quad \text{customer} = \text{none} \land \text{customer}' \neq \text{none} \land \\
&\quad \text{only}(\text{customer}).
\end{align*}
\]
Transitions: Symbolic Representation

\[
\begin{align*}
st_{\text{coffee}} & \quad \{0, 1\} \\
st_{\text{beer}} & \quad \{0, 1\} \\
disp & \quad \{\text{none, beer, coffee}\} \\
coins & \quad \{0, 1, 2, 3\} \\
customer & \quad \{\text{none, student, prof}\}
\end{align*}
\]

\[
\begin{align*}
\text{Recharge} & \quad \text{def} \quad \text{customer} = \text{none} \land \\
& \quad \text{st}_{\text{coffee}}' \land \text{st}_{\text{beer}}' \land \\
& \quad \text{only(st}_{\text{coffee}}, \text{st}_{\text{beer}}).
\end{align*}
\]

\[
\begin{align*}
\text{Customer\_arrives} & \quad \text{def} \quad \text{customer} = \text{none} \land \text{customer}' \neq \text{none} \land \\
& \quad \text{only(customer)}
\end{align*}
\]

\[
\begin{align*}
\text{Customer\_leaves} & \quad \text{def} \quad \text{customer} \neq \text{none} \land \text{customer}' = \text{none} \land \\
& \quad \text{only(customer)}.
\end{align*}
\]
Transitions: Symbolic Representation

\[
\begin{align*}
\text{st\_coffee} & \{0, 1\} \\
\text{st\_beer} & \{0, 1\} \\
\text{disp} & \{\text{none, beer, coffee}\} \\
\text{coins} & \{0, 1, 2, 3\} \\
\text{customer} & \{\text{none, student, prof}\}
\end{align*}
\]

\[
\begin{align*}
\text{Recharge} & \equiv \text{customer} = \text{none} \land \\
& \text{st\_coffee}' \land \text{st\_beer}' \land \\
& \text{only}(\text{st\_coffee, st\_beer}).
\end{align*}
\]

\[
\begin{align*}
\text{Customer\_arrives} & \equiv \text{customer} = \text{none} \land \text{customer}' \neq \text{none} \land \\
& \text{only}(\text{customer})
\end{align*}
\]

\[
\begin{align*}
\text{Customer\_leaves} & \equiv \text{customer} \neq \text{none} \land \text{customer}' = \text{none} \land \\
& \text{only}(\text{customer}).
\end{align*}
\]
Transitions: Symbolic Representation

\[
\begin{align*}
st_{\text{coffee}} &\{0, 1\} \\
st_{\text{beer}} &\{0, 1\} \\
disp &\\{\text{none, beer, coffee}\} \\
coins &\{0, 1, 2, 3\} \\
customer &\\{\text{none, student, prof}\}
\end{align*}
\]

\[
\begin{align*}
\text{Recharge} &\equiv \text{customer} = \text{none} \land
st_{\text{coffee}}' \land st_{\text{beer}}' \land
\text{only}(st_{\text{coffee}}, st_{\text{beer}}). \\
\text{Customer\_arrives} &\equiv \text{customer} = \text{none} \land \text{customer}' \neq \text{none} \land
\text{only}(\text{customer}) \\
\text{Customer\_leaves} &\equiv \text{customer} \neq \text{none} \land \text{customer}' = \text{none} \land
\text{only}(\text{customer}). \\
\text{Coin\_insert} &\equiv \text{customer} \neq \text{none} \land \text{coins} \neq 3 \land
(coins = 0 \rightarrow coins' = 1) \land
(coins = 1 \rightarrow coins' = 2) \land
(coins = 2 \rightarrow coins' = 3) \land
\text{only}(\text{coins}).
\end{align*}
\]
## Transitions

<table>
<thead>
<tr>
<th>State</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>st_coffee</td>
<td>{0, 1}</td>
</tr>
<tr>
<td>st_beer</td>
<td>{0, 1}</td>
</tr>
<tr>
<td>disp</td>
<td>{none, beer, coffee}</td>
</tr>
<tr>
<td>coins</td>
<td>{0, 1, 2, 3}</td>
</tr>
<tr>
<td>customer</td>
<td>{none, student, prof}</td>
</tr>
</tbody>
</table>

---

```
Dispense\_beer
Dispense\_coffee
Take\_drink
```
Transitions

st\_coffee \{0, 1\}
st\_beer \{0, 1\}
disp \{none, beer, coffee\}
coins \{0, 1, 2, 3\}
customer \{none, student, prof\}

\textbf{Dispense\_beer} \quad \textbf{Dispense\_coffee} \quad \textbf{Take\_drink}

\textbf{Dispense\_beer} \ \overset{\text{def}}{=} \ \text{customer} = \text{student} \land \text{st\_beer} \land \\
\text{disp} = \text{none} \land (\text{coins} = 2 \lor \text{coins} = 3) \land \\
\text{disp}^\prime = \text{beer} \land \\
(\text{coins} = 2 \rightarrow \text{coins}^\prime = 0) \land \\
(\text{coins} = 3 \rightarrow \text{coins}^\prime = 1) \land \\
only(\text{st\_beer, disp, coins}).
Transitions

\begin{align*}
st\_coffee & \{0, 1\} \\
st\_beer & \{0, 1\} \\
disp & \{none, beer, coffee\} \\
coins & \{0, 1, 2, 3\} \\
customer & \{none, student, prof\}
\end{align*}

\begin{align*}
\textit{Dispense\_beer} & \overset{\text{def}}{=} \\
\quad & \text{customer} = student \land st\_beer \land \\
\quad & disp = none \land (coins = 2 \lor coins = 3) \land \\
\quad & disp' = beer \land \\
\quad & (coins = 2 \rightarrow coins' = 0) \land \\
\quad & (coins = 3 \rightarrow coins' = 1) \land \\
\quad & only(st\_beer, disp, coins).
\end{align*}
## Transitions

<table>
<thead>
<tr>
<th>State</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>st_coffee</code></td>
<td>{0, 1}</td>
</tr>
<tr>
<td><code>st_beer</code></td>
<td>{0, 1}</td>
</tr>
<tr>
<td><code>disp</code></td>
<td>{none, beer, coffee}</td>
</tr>
<tr>
<td><code>coins</code></td>
<td>{0, 1, 2, 3}</td>
</tr>
<tr>
<td><code>customer</code></td>
<td>{none, student, prof}</td>
</tr>
</tbody>
</table>

\[
\text{\textit{Dispense\_beer}} \overset{\text{def}}{=} \begin{aligned}
\text{customer} &= \text{student} \land \text{st\_beer} \land \\
\text{disp} &= \text{none} \land (\text{coins} = 2 \lor \text{coins} = 3) \land \\
\text{disp}' &= \text{beer} \land \\
(\text{coins} = 2 \rightarrow \text{coins}' = 0) \land \\
(\text{coins} = 3 \rightarrow \text{coins}' = 1) \land \\
\text{only}(\text{st\_beer, disp, coins}).
\end{aligned}
\]

\[
\text{\textit{Dispense\_coffee}} \overset{\text{def}}{=} \begin{aligned}
\text{customer} &= \text{prof} \land \text{st\_coffee} \land \\
\text{disp} &= \text{none} \land \text{coins} \neq 0 \land \\
\text{disp}' &= \text{coffee} \land \\
(\text{coins} = 1 \rightarrow \text{coins}' = 0) \land \\
(\text{coins} = 2 \rightarrow \text{coins}' = 1) \land \\
(\text{coins} = 3 \rightarrow \text{coins}' = 2) \land \\
\text{only}(\text{st\_coffee, disp, coins}).
\end{aligned}
\]

\[
\text{\textit{Take\_drink}} \overset{\text{def}}{=} \begin{aligned}
\text{customer} &= \text{none} \land \text{disp} = \text{none} \land \\
\text{disp}' &= \text{none} \land \\
\text{only}(\text{disp}).
\end{aligned}
\]

\[
\text{\textit{Dispense\_coffee}} \overset{\text{def}}{=} \begin{aligned}
\text{customer} &= \text{prof} \land \text{st\_coffee} \land \\
\text{disp} &= \text{none} \land \text{coins} \neq 0 \land \\
\text{disp}' &= \text{coffee} \land \\
(\text{coins} = 1 \rightarrow \text{coins}' = 0) \land \\
(\text{coins} = 2 \rightarrow \text{coins}' = 1) \land \\
(\text{coins} = 3 \rightarrow \text{coins}' = 2) \land \\
\text{only}(\text{st\_coffee, disp, coins}).
\end{aligned}
\]
Transitions

\[
\begin{align*}
st\_coffee & \quad \{0, 1\} \\
st\_beer & \quad \{0, 1\} \\
disp & \quad \{none, beer, coffee\} \\
coins & \quad \{0, 1, 2, 3\} \\
customer & \quad \{none, student, prof\}
\end{align*}
\]

\[
\begin{align*}
\text{Dispense\_beer} & \quad \overset{\text{def}}{=} \text{customer} = student \wedge st\_beer \wedge \\
& \quad \text{disp} = none \wedge (coins = 2 \lor coins = 3) \wedge \\
& \quad \text{disp}^\prime = beer \wedge \\
& \quad (coins = 2 \rightarrow coins^\prime = 0) \wedge \\
& \quad (coins = 3 \rightarrow coins^\prime = 1) \wedge \\
& \quad only(st\_beer, disp, coins).
\end{align*}
\]

\[
\begin{align*}
\text{Dispense\_coffee} & \quad \overset{\text{def}}{=} \text{customer} = prof \wedge st\_coffee \wedge \\
& \quad \text{disp} = none \wedge coins \neq 0 \wedge \\
& \quad \text{disp}^\prime = coffee \wedge \\
& \quad (coins = 1 \rightarrow coins^\prime = 0) \wedge \\
& \quad (coins = 2 \rightarrow coins^\prime = 1) \wedge \\
& \quad (coins = 3 \rightarrow coins^\prime = 2) \wedge \\
& \quad only(st\_coffee, disp, coins).
\end{align*}
\]
Transitions

\[
\begin{align*}
\text{st\_coffee} & \{0, 1\} \\
\text{st\_beer} & \{0, 1\} \\
\text{disp} & \{\text{none, beer, coffee}\} \\
\text{coins} & \{0, 1, 2, 3\} \\
\text{customer} & \{\text{none, student, prof}\}
\end{align*}
\]

\[\begin{align*}
\text{Dispense}\_\text{beer} & \equiv \text{customer} = \text{student} \land \text{st\_beer} \land \\
& \text{disp} = \text{none} \land (\text{coins} = 2 \lor \text{coins} = 3) \land \\
& \text{disp}' = \text{beer} \land \\
& (\text{coins} = 2 \rightarrow \text{coins}' = 0) \land \\
& (\text{coins} = 3 \rightarrow \text{coins}' = 1) \land \\
& \text{only}(\text{st\_beer, disp, coins}).
\end{align*}\]

\[\begin{align*}
\text{Dispense}\_\text{coffee} & \equiv \text{customer} = \text{prof} \land \text{st\_coffee} \land \\
& \text{disp} = \text{none} \land \text{coins} \neq 0 \land \\
& \text{disp}' = \text{coffee} \land \\
& (\text{coins} = 1 \rightarrow \text{coins}' = 0) \land \\
& (\text{coins} = 2 \rightarrow \text{coins}' = 1) \land \\
& (\text{coins} = 3 \rightarrow \text{coins}' = 2) \land \\
& \text{only}(\text{st\_coffee, disp, coins}).
\end{align*}\]

\[\begin{align*}
\text{Take}\_\text{drink} & \equiv \text{customer} \neq \text{none} \land \text{disp} \neq \text{none} \land \\
& \text{disp}' = \text{none} \land \\
& \text{only}(\text{disp}).
\end{align*}\]
Transitions

Model checkers often use the convention that the variables that can change are those variables $x$ such that $x'$ occurs in the problem. Under this convention we can remove only(...) from all transitions and change `Dispense_beer` and `Dispense_coffee` as follows:

\[
\text{Dispense_beer} \overset{\text{def}}{=} \begin{align*}
\text{customer} &= \text{student} \land \text{st\_beer} \land \\
\text{disp} &= \text{none} \land (\text{coins} = 2 \lor \text{coins} = 3) \land \\
\text{disp}' &= \text{beer} \land \\
(\text{coins} = 2 \rightarrow \text{coins}' = 0) \land \\
(\text{coins} = 3 \rightarrow \text{coins}' = 1) \land \\
\text{st\_beer}' &= \text{st\_beer}'.
\end{align*}
\]

\[
\text{Dispense_coffee} \overset{\text{def}}{=} \begin{align*}
\text{customer} &= \text{prof} \land \text{st\_coffee} \land \\
\text{disp} &= \text{none} \land \text{coins} \neq 0 \land \\
\text{disp}' &= \text{coffee} \land \\
(\text{coins} = 1 \rightarrow \text{coins}' = 0) \land \\
(\text{coins} = 2 \rightarrow \text{coins}' = 1) \land \\
(\text{coins} = 3 \rightarrow \text{coins}' = 2) \land \\
\text{st\_coffee}' &= \text{st\_coffee}'.
\end{align*}
\]
1. There is no state in which professor and student are both customers.
Temporal Properties of Transition Systems

1. There is no state in which professor and student are both customers.
2. Students never drink coffee.
Temporal Properties of Transition Systems

1. There is no state in which professor and student are both customers.
2. Students never drink coffee.
3. The machine cannot dispense drinks forever without recharging.