

Exercise 1 (problem 1)

The following formula has its parentheses removed. Restore the parentheses.

$$\neg p_1 \rightarrow \neg \neg p_2 \leftrightarrow p_3 \wedge p_4.$$

$$((\neg p_1) \rightarrow (\neg(\neg p_2))) \leftrightarrow (p_3 \wedge p_4)$$

Connective	Name	Priority
\top	verum	
\perp	falsum	
\neg	negation	4
\wedge	conjunction	3
\vee	disjunction	3
\rightarrow	implication	2
\leftrightarrow	equivalence	1

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Typical Errors

- ▶ Reverse order of connectives.
- ▶ Putting parentheses around variables, for example $\neg p_1$.
- ▶ Some parentheses missing;
- ▶ The formula $\neg\neg p_2$ replaced by p_2 .

Exercise 1 (problem 2)

Show that the formulas $p \rightarrow (q \rightarrow r)$ and $(p \rightarrow q) \rightarrow r$ are **not equivalent** by finding an interpretation in which they have different truth values.

Take the interpretation $I_1 = \{p \mapsto 0, q \mapsto 0, r \mapsto 0\}$. We have

$$\begin{aligned}I_1(p \rightarrow (q \rightarrow r)) &= 1; \\I_1((p \rightarrow q) \rightarrow r) &= 0.\end{aligned}$$

Another solution is the interpretation $I_2 = \{p \mapsto 0, q \mapsto 1, r \mapsto 0\}$. We have

$$\begin{aligned}I_2(p \rightarrow (q \rightarrow r)) &= 1; \\I_2((p \rightarrow q) \rightarrow r) &= 0.\end{aligned}$$

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Typical errors

- ▶ A general misunderstanding of what an interpretation is. For example, claiming that $p \rightarrow (q \rightarrow \neg r)$ is an answer.
- ▶ Incomplete simplifications when evaluating formulas in interpretations. For example some did not simplify formulas $\top \rightarrow \perp$. Some went even further and decided that $\perp \rightarrow \top$ and $\top \rightarrow \top$ give non-equivalent formulas.
- ▶ Too many errors with evaluation implications: check the definition of truth and the operation table for \rightarrow .
- ▶ Some do not understand when equivalence/non-equivalence of formulas mean.
- ▶ The question was to give interpretations. Some built correct truth tables but did not give interpretations. Some correctly noted that p and r should have the value 0 but did not give any interpretation.