

# Outline

## Exercise 5

Problem 1

Problem 2

Problem 3

## Exercise 5 (problem 1)

Consider the set consisting of the following clauses:

$$\neg p_0 \vee \neg p_1 \vee \neg p_2, \quad p_0 \vee \neg p_2, \quad \neg p_0 \vee p_1, \quad p_1 \vee p_2, \quad \neg p_0 \vee \neg p_1 \vee p_2.$$

Show how WSAT can find a model of this set starting with the initial random interpretation  $\{p_0 \mapsto 1, p_1 \mapsto 0, p_2 \mapsto 1\}$ .

# Solution

	1		0		1
$\neg p_0$	∨	$\neg p_1$	∨	$\neg p_2$	
$p_0$			∨	$\neg p_2$	
$\neg p_0$	∨	$p_1$			
		$p_1$	∨	$p_2$	
$\neg p_0$	∨	$\neg p_1$	∨	$p_2$	

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped atom
	$p_0$	$p_1$	$p_2$			
1	1	0	1	$\neg p_0 \vee p_1$	$p_0, p_1$	$p_0$
2	0	0	1	$p_0 \vee \neg p_2$	$p_0, p_2$	
3	0	0	0	$p_1 \vee p_2$	$p_1, p_2$	
	0	1	0			

# Solution

$0$	$\vee$	$0$	$\vee$	$1$
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	$\neg p_2$
$p_0$			$\vee$	$\neg p_2$
$\neg p_0$	$\vee$	$p_1$		
		$p_1$	$\vee$	$p_2$
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	$p_2$

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	$p_0$	$p_1$	$p_2$			
1	1	0	1	$\neg p_0 \vee p_1$	$p_0, p_1$	$p_0$
2	0	0	1	$p_0 \vee \neg p_2$	$p_0, p_2$	$p_2$
3	0	0	0	$p_1 \vee p_2$	$p_1, p_2$	
	0	1	0			

# Solution

0		0		0
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	$\neg p_2$
$p_0$			$\vee$	$\neg p_2$
$\neg p_0$	$\vee$	$p_1$		
		$p_1$	$\vee$	$p_2$
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	$p_2$

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped atom
	$p_0$	$p_1$	$p_2$			
1	1	0	1	$\neg p_0 \vee p_1$	$p_0, p_1$	$p_0$
2	0	0	1	$p_0 \vee \neg p_2$	$p_0, p_2$	$p_2$
3	0	0	0	$p_1 \vee p_2$	$p_1, p_2$	$p_1$
	0	1	0			

# Solution

0		1		0
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	$\neg p_2$
$p_0$			$\vee$	$\neg p_2$
$\neg p_0$	$\vee$	$p_1$		
		$p_1$	$\vee$	$p_2$
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	$p_2$

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	$p_0$	$p_1$	$p_2$			
1	1	0	1	$\neg p_0 \vee p_1$	$p_0, p_1$	$p_0$
2	0	0	1	$p_0 \vee \neg p_2$	$p_0, p_2$	$p_2$
3	0	0	0	$p_1 \vee p_2$	$p_1, p_2$	$p_1$
	0	1	0			

# Typical errors

- ▶ Some (few) mistakes in finding out which clauses are unsatisfiable.

## Exercise 5 (problem 2)

Consider the set consisting of the following clauses:

$$\begin{array}{cccc} p_0 \vee \neg p_1 \vee p_2 & p_0 \vee \neg p_1 \vee p_2 \vee p_4 & \neg p_0 \vee p_1 \vee \neg p_2 & \neg p_0 \vee \neg p_1 \vee \neg p_2 \vee \neg p_4 \\ p_0 \vee \neg p_1 \vee p_4 & p_3 \vee p_2 \vee p_4 \vee \neg p_0 & \neg p_2 \vee \neg p_2 \vee p_4 \vee p_3 & \neg p_2 \vee \neg p_0 \vee p_4 \vee p_4 \\ p_0 \vee p_3 \vee \neg p_4 & p_0 \vee \neg p_1 \vee \neg p_2 \vee \neg p_3 & \neg p_1 \vee \neg p_2 \vee \neg p_3 & p_1 \vee \neg p_2 \vee \neg p_3 \vee \neg p_4 \\ p_1 \vee p_2 & p_2 \vee p_3 \vee \neg p_4 \vee p_3 & \neg p_0 \vee \neg p_2 \vee \neg p_3 \vee \neg p_4 & p_0 \vee p_2 \vee p_4 \end{array}$$

For each of the variables  $p_0, p_1, p_2, p_3, p_4$  find the probability that WSAT will choose this variable for flipping when the current interpretation is  $\{p_0 \mapsto 0, p_1 \mapsto 0, p_2 \mapsto 0, p_3 \mapsto 0, p_4 \mapsto 0\}$ .

# Solution

First of all, we should select clauses that are false in the current interpretation. These are only two such clauses shown in the left column of the following table.

clause	$p_0$	$p_1$	$p_2$	$p_3$	$p_4$
$p_1 \vee p_2$	0	$\frac{1}{4}$	$\frac{1}{4}$	0	0
$p_0 \vee p_2 \vee p_4$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$
sum	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{5}{12}$	0	$\frac{1}{6}$

In each cell of the table corresponding to a clause and variable  $p_i$  we write the probability that this clause and then this variable is selected. Note that the probability of selecting each clause is the same, that is  $\frac{1}{2}$ .

The bottom row on the table gives the answer, which is the sum of probabilities of selecting this variable over all clauses.

# Typical errors

- ▶ A vast majority of students do not calculate probabilities properly. For example, the probability is calculated according to the number of occurrences of each variable in unsatisfiable clauses. This is wrong since the way WSAT works is the following: first one of the candidate clauses is selected with an equal probability; then a variable in this clause is selected, again with an equal probability.
- ▶ There are some other miscalculations.

## Exercise 5 (problem 3)

Show validity of the following formula using semantic tableaux:

$$(p \rightarrow r) \rightarrow (p \vee q \rightarrow r \vee q).$$

We will use the following characterisation of validity: a formula  $A$  is valid iff there is a closed tableau for  $A = 0$ . Such a tableau is shown on the next slide.

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## Solution

A process of building a closed tableau is illustrated in the following slides. At each step we highlight the signed formula or formulas to which branch expansion or branch closure is applied and the new signed formulas added to the tableau.

$$((p \rightarrow r) \rightarrow (p \vee q \rightarrow r \vee q)) = 0$$

$$\begin{array}{c} | \\ (p \rightarrow r) = 1 \\ (p \vee q \rightarrow r \vee q) = 0 \end{array}$$

$$\begin{array}{c} | \\ (p \vee q) = 1 \\ (r \vee q) = 0 \end{array}$$

$$\begin{array}{c} | \\ r = 0 \\ q = 0 \end{array}$$

$$\begin{array}{c} / \qquad \backslash \\ p = 1 \qquad q = 1 \\ \qquad \qquad \text{(closed)} \\ / \qquad \backslash \\ p = 0 \qquad r = 1 \\ \text{(closed)} \qquad \text{(closed)} \end{array}$$

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$$\begin{array}{l} r = 0 \\ q = 0 \end{array}$$

$$p = 1$$

$$q = 1 \\ \text{(closed)}$$

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# Typical errors

- ▶ Some applied splitting on variables instead of tableau rules.
- ▶ Some do not understand how to use tableaux for checking validity of formulas.
- ▶ There were all kinds of errors in the application tableau expansion rules. For example, some applying expansion rule to two formulas at once and put the results on separate branches while only one rule can be applied at a time and only on a single branch.