GSAT with random walks

procedure *GSATwithWalks*(S)
input: set of clauses S
output: interpretation I such that $I \models S$ or *don't know*
parameters: integers *MAX-TRIES*, *MAX-FLIPS*
real number $0 \leq \pi \leq 1$ (probability of a sideways move),

begin
repeat *MAX-TRIES* times
I := random interpretation ;
if $I \models S$ **then** **return** I
repeat *MAX-FLIPS* times
with probability π
p := an atom such that *flip*(I, p) satisfies the maximal number of clauses in S
with probability $1 - \pi$
randomly select p among atoms occurring in clauses false in I
I = *flip*(I, p) ;
if $I \models S$ **then** **return** I
return *don't know*
end

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begin
repeat *MAX-TRIES* times
I := random interpretation ;
if $I \models S$ **then return** I
repeat *MAX-FLIPS* times
with probability π
p := an atom such that $flip(I, p)$ satisfies
the maximal number of clauses in S
with probability $1 - \pi$
randomly select p among atoms occurring in clauses false in I
$I = flip(I, p)$;
if $I \models S$ **then return** I
return *don't know*
end

GSAT with random walks

```
procedure GSATwithWalks(S)
input: set of clauses S
output: interpretation I such that I ⊨ S or don't know
parameters: integers MAX-TRIES, MAX-FLIPS
            real number 0 ≤ π ≤ 1 (probability of a sideways move),
begin
 repeat MAX-TRIES times
  I := random interpretation ;
  if I ⊨ S then return I
  repeat MAX-FLIPS times
   with probability π
    p := an atom such that flip(I, p) satisfies
         the maximal number of clauses in S
   with probability 1 − π
    randomly select p among atoms occurring in clauses false in I
   I = flip(I, p) ;
   if I ⊨ S then return I
 return don't know
end
```

WSAT

procedure *WSAT*(S)
input: set of clauses S
output: interpretation I such that $I \models S$ or *don't know*
parameters: integers *MAX-TRIES*, *MAX-FLIPS*
begin
  repeat *MAX-TRIES* times
    $I := $ random interpretation
    if $I \models S$ **then return** I
    repeat *MAX-FLIPS* times
      randomly select a clause $C \in S$ such that $I \not\models S$
      randomly select an atom p in C
      $I = flip(I, p)$
      if $I \models S$ **then return** I
  return *don't know*
end

WSAT

```
procedure WSAT(S)
input: set of clauses S
output: interpretation I such that I ⊨ S or don't know
parameters: integers MAX-TRIES, MAX-FLIPS
begin
  repeat MAX-TRIES times
    I := random interpretation
    if I ⊨ S then return I
    repeat MAX-FLIPS times
      randomly select a clause C ∈ S such that I ⊭ S
      randomly select an atom p in C
      I = flip(I, p)
      if I ⊨ S then return I
  return don't know
end
```

WSAT

```
procedure WSAT(S)
input: set of clauses S
output: interpretation I such that I ⊨ S or don't know
parameters: integers MAX-TRIES, MAX-FLIPS
begin
 repeat MAX-TRIES times
  I := random interpretation
  if I ⊨ S then return I
  repeat MAX-FLIPS times
   randomly select a clause C ∈ S such that I ⊭ S
   randomly select an atom p in C
   I = flip(I, p)
   if I ⊨ S then return I
 return don't know
end
```

WSAT example

0		1		1
p_1	$\vee$	$\neg p_2$	$\vee$	p_3
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped atom
	p_1	p_2	p_3			
1	0	0	1	$p_1 \vee p_2$	p_1, p_2	p_1
2	1	0	1	$\neg p_1 \vee p_2$		
3	1	1	1	$\neg p_1 \vee \neg p_3$		
	1	1	0			

WSAT example

0		1		1
p_1	$\vee$	$\neg p_2$	$\vee$	p_3
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

flip no.	interpretation p_1	p_2	p_3	unsatisfied clauses	candidates for flipping	flipped atom
1	0	0	1	$p_1 \vee p_2$	p_1, p_2	p_1
2	1	0	1	$\neg p_1 \vee \neg p_3$	p_1, p_2, p_3	
				$\neg p_1 \vee p_2$		
3	1	1	1	$\neg p_2 \vee \neg p_3$	p_1, p_2, p_3	
				$\neg p_1 \vee \neg p_3$		
	1	1	0			

WSAT example

$$
\begin{array}{ccccc}
1 & & 1 & & 1 \\
\hline
p_1 & \vee & \neg p_2 & \vee & p_3 \\
 & & \neg p_2 & \vee & \neg p_3 \\
\neg p_1 & & & \vee & \neg p_3 \\
\neg p_1 & \vee & p_2 & & \\
p_1 & \vee & p_2 & &
\end{array}
$$

flip no.	interpretation p_1	p_2	p_3	unsatisfied clauses	candidates for flipping	flipped atom
1	0	0	1	$p_1 \vee p_2$	p_1, p_2	p_1
2	1	0	1	$\neg p_1 \vee \neg p_3$ $\neg p_1 \vee p_2$	p_1, p_2, p_3	p_2
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	1	1	0			

WSAT example

p_1		p_2		p_3
1		1		1
p_1	$\vee$	$\neg p_2$	$\vee$	p_3
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

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3	1	1	1	$\neg p_2 \vee \neg p_3$ $\neg p_1 \vee \neg p_3$	p_1, p_2, p_3	
	1	1	0			

WSAT example

1		1		1
p_1	$\vee$	$\neg p_2$	$\vee$	p_3
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

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3	1	1	1	$\neg p_2 \vee \neg p_3$ $\neg p_1 \vee \neg p_3$	p_1, p_2, p_3	p_3
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WSAT example

1		1		1
p_1	$\vee$	$\neg p_2$	$\vee$	p_3
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

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3	1	1	1	$\neg p_2 \vee \neg p_3$ $\neg p_1 \vee \neg p_3$	p_1, p_2, p_3	p_3
	1	1	0			

WSAT example

1		1		0
p_1	$\vee$	$\neg p_2$	$\vee$	p_3
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

flip no.	interpretation p_1	p_2	p_3	unsatisfied clauses	candidates for flipping	flipped atom
1	0	0	1	$p_1 \vee p_2$	p_1, p_2	p_1
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3	1	1	1	$\neg p_2 \vee \neg p_3$ $\neg p_1 \vee \neg p_3$	p_1, p_2, p_3	p_3
	1	1	0			

WSAT example

1		1		0
p_1	$\vee$	$\neg p_2$	$\vee$	p_3
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	p_2		
p_1	$\vee$	p_2		

flip no.	interpretation p_1	p_2	p_3	unsatisfied clauses	candidates for flipping	flipped atom
1	0	0	1	$p_1 \vee p_2$	p_1, p_2	p_1
2	1	0	1	$\neg p_1 \vee \neg p_3$ $\neg p_1 \vee p_2$	p_1, p_2, p_3	p_2
3	1	1	1	$\neg p_2 \vee \neg p_3$ $\neg p_1 \vee \neg p_3$	p_1, p_2, p_3	p_3
	1	1	0			

Outline

Semantic Tableaux

Signed Formula

- Signed formula: an expression $A = b$, where A is a formula and b a boolean value.
- A signed formula $A = b$ is true in an interpretation I, denoted by $I \models A = b$, if $I(A) = b$.
- If $A = b$ is true in I, we also say that I is a model of $A = b$, or that I satisfies $A = b$.
- A signed formula is satisfiable if it has a model.

Note:

1. For every formula A and interpretation I exactly one of the signed formulas $A = 1$ and $A = 0$ is true in I.
2. A formula A is satisfiable if and only if so is the signed formula $A = 1$.

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How to find a model of a signed formula?

Operation table for $\rightarrow$:

$\rightarrow$	$B = 1$	$B = 0$
$A = 1$	1	0
$A = 0$	1	1

$\rightarrow$	$B = 1$	$B = 0$
$A = 1$	1	0
$A = 0$	1	1

Example: $(A \rightarrow B) = 1$.

So $(A \rightarrow B) = 1$ if and only if $A = 0$ OR $B = 1$.

Likewise, $(A \rightarrow B) = 0$ if and only if $A = 1$ AND $B = 0$.

So we can use AND-OR trees to carry out case analysis.

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How to find a model of a signed formula?

Operation table for $\to$:

$\to$	$B = 1$	$B = 0$
$A = 1$	1	0
$A = 0$	1	1

$\to$	$B = 1$	$B = 0$
$A = 1$	1	0
$A = 0$	1	1

Example: $(A \to B) = 1$.

So $(A \to B) = 1$ if and only if $A = 0$ OR $B = 1$.

Likewise, $(A \to B) = 0$ if and only if $A = 1$ AND $B = 0$.

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How to find a model of a signed formula?

Operation table for $\rightarrow$:

$\rightarrow$	$B = 1$	$B = 0$
$A = 1$	1	0
$A = 0$	1	1

$\rightarrow$	$B = 1$	$B = 0$
$A = 1$	1	0
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So we can use AND-OR trees to carry out case analysis.

Tableau

Tableau: a tree having signed formulas at nodes.

Tableau for a signed formula $A = b$ has $A = b$ as a root.

Alternatively, we can regard a tableau as a collection of branches; each branch is a set of signed formulas.

Notation for branches: $A_1 = b_1 \mid \ldots \mid A_n = b_n$.

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Notation for branches: $A_1 = b_1 \mid \dots \mid A_n = b_n$.

Branch Expansion Rules

$$
\begin{array}{rcl}
(A_1 \wedge \ldots \wedge A_n) = 0 & \rightsquigarrow & A_1 = 0 \mid \ldots \mid A_n = 0 \\
(A_1 \wedge \ldots \wedge A_n) = 1 & \rightsquigarrow & A_1 = 1, \ldots, A_n = 1 \\
\\
(A_1 \vee \ldots \vee A_n) = 0 & \rightsquigarrow & A_1 = 0, \ldots, A_n = 0 \\
(A_1 \vee \ldots \vee A_n) = 1 & \rightsquigarrow & A_1 = 1 \mid \ldots \mid A_n = 1 \\
\\
(A_1 \rightarrow A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(A_1 \rightarrow A_2) = 1 & \rightsquigarrow & A_1 = 0 \mid A_2 = 1 \\
\\
(\neg A_1) = 0 & \rightsquigarrow & A_1 = 1 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0 \\
\\
(A_1 \leftrightarrow A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 1 \mid A_1 = 1, A_2 = 0 \\
(A_1 \leftrightarrow A_2) = 1 & \rightsquigarrow & A_1 = 0, A_2 = 0 \mid A_1 = 1, A_2 = 1
\end{array}
$$

Branch Closure Rules

These rules are introduced to mark when the set of signed formulas on a branch is unsatisfiable.

A branch is marked closed in any of the following cases:

- it contains both $p = 0$ and $p = 1$ for some atom p
- it contains $\top = 0$;
- it contains $\bot = 1$.

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- it contains $\top = 0$;
- it contains $\bot = 1$.

A Semantic Tableau

$$(\neg(q \vee p \rightarrow p \vee q)) = 1$$

$$(q \vee p \rightarrow p \vee q) = 0$$

$$(q \vee p) = 1$$
$$(p \vee q) = 0$$

$$p = 0$$
$$q = 0$$

$q = 1$	$p = 1$
closed	closed

$$\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \rightarrow A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}$$

A Semantic Tableau

$$(\neg(q \vee p \to p \vee q)) = 1$$

$$(q \vee p \to p \vee q) = 0$$

$$(q \vee p) = 1$$
$$(p \vee q) = 0$$

$$p = 0$$
$$q = 0$$

$q = 1$ closed | $p = 1$ closed

$$(A_1 \vee A_2) = 0 \quad \rightsquigarrow \quad A_1 = 0, A_2 = 0$$
$$(A_1 \vee A_2) = 1 \quad \rightsquigarrow \quad A_1 = 1 \mid A_2 = 1$$
$$(A_1 \to A_2) = 0 \quad \rightsquigarrow \quad A_1 = 1, A_2 = 0$$
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$$(\neg(q \vee p \rightarrow p \vee q)) = 1$$

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$$(p \vee q) = 0$$

$$p = 0$$
$$q = 0$$

$q = 1$ closed | $p = 1$ closed

$$
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(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
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$$(\neg(q \vee p \rightarrow p \vee q)) = 1$$

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A Semantic Tableau

$$(\neg(q \vee p \rightarrow p \vee q)) = 1$$

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$$(q \vee p) = 1$$
$$(p \vee q) = 0$$

$$p = 0$$
$$q = 0$$

$q = 1$ closed | $p = 1$ closed

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A Semantic Tableau

$$(\neg(q \vee p \rightarrow p \vee q)) = 1$$

$$(q \vee p \rightarrow p \vee q) = 0$$

$$(q \vee p) = 1$$
$$(p \vee q) = 0$$

$$p = 0$$
$$q = 0$$

$q = 1$	$p = 1$
closed	closed

$$
\begin{aligned}
(A_1 \vee A_2) = 0 &\quad \rightsquigarrow \quad A_1 = 0, A_2 = 0 \\
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(\neg A_1) = 1 &\quad \rightsquigarrow \quad A_1 = 0
\end{aligned}
$$

A Semantic Tableau

$$(\neg(q \vee p \rightarrow p \vee q)) = 1$$

$$(q \vee p \rightarrow p \vee q) = 0$$

$$(q \vee p) = 1$$
$$(p \vee q) = 0$$

$$p = 0$$
$$q = 0$$

$q = 1$ closed | $p = 1$ closed

$$(A_1 \vee A_2) = 0 \quad \rightsquigarrow \quad A_1 = 0, A_2 = 0$$
$$(A_1 \vee A_2) = 1 \quad \rightsquigarrow \quad A_1 = 1 \mid A_2 = 1$$
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$$(\neg A_1) = 1 \quad \rightsquigarrow \quad A_1 = 0$$

A Semantic Tableau

$$(\neg(q \vee p \rightarrow p \vee q)) = 1$$

$$(q \vee p \rightarrow p \vee q) = 0$$

$$(q \vee p) = 1$$
$$(p \vee q) = 0$$

$$p = 0$$
$$q = 0$$

$$q = 1 \qquad\qquad p = 1$$

closed closed

$$\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \rightarrow A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}$$

A Semantic Tableau

$$(\neg(q \vee p \to p \vee q)) = 1$$

$$(q \vee p \to p \vee q) = 0$$

$$(q \vee p) = 1$$
$$(p \vee q) = 0$$

$$p = 0$$
$$q = 0$$

$q = 1$	$p = 1$
closed	closed

$$
\begin{array}{rcl}
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\end{array}
$$

A Semantic Tableau

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$$p = 0$$
$$q = 0$$

$q = 1$ | $p = 1$
closed | closed

$$(A_1 \vee A_2) = 0 \quad \rightsquigarrow \quad A_1 = 0, A_2 = 0$$
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A Semantic Tableau

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$$(p \vee q) = 0$$

$$p = 0$$
$$q = 0$$

$q = 1$	$p = 1$
closed	closed

$$\begin{aligned}
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\end{aligned}$$

Finding Models Using Tableaux

$$(\neg((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r))) = 1$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r) \rightarrow (\neg p \rightarrow r)) = 0$$

$$((p \rightarrow q) \wedge (p \wedge q \rightarrow r)) = 1$$
$$(\neg p \rightarrow r) = 0$$

$$(p \rightarrow q) = 1$$
$$(p \wedge q \rightarrow r) = 1$$

$$(\neg p) = 1$$
$$r = 0$$

$p = 0$ | $q = 1$

$p = 0$

$(p \wedge q) = 0$ | $r = 1$

$p = 0$ | $q = 0$

Build an open branch on which all rules have been applied.

Select signed atoms on this branch

They give us a model

$$\{r \mapsto 0, p \mapsto 0, q \mapsto \cdots\}$$

Finding Models Using Tableaux

$$(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1$$

$$((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0$$

$$((p \to q) \land (p \land q \to r)) = 1$$
$$(\neg p \to r) = 0$$

$$(p \to q) = 1$$
$$(p \land q \to r) = 1$$

$$(\neg p) = 1$$
$$r = 0$$

$$p = 0 \qquad q = 1$$

$$p = 0$$

$$(p \land q) = 0 \qquad r = 1$$

$$p = 0 \qquad q = 0$$

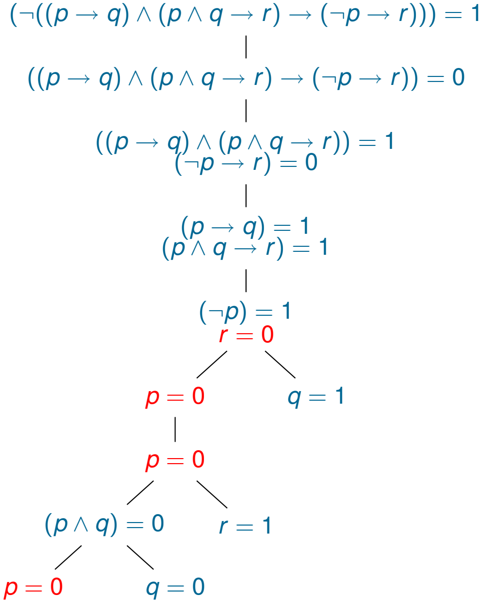
Build an open branch on which all rules have been applied.

Select signed atoms on this branch

They give us a model

$$\{r \mapsto 0, p \mapsto 0, q \mapsto \cdots\}$$

Finding Models Using Tableaux



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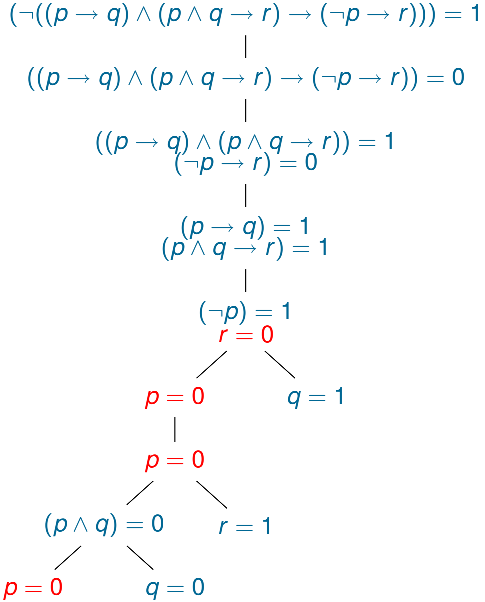
Finding Models Using Tableaux



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$$\{r \mapsto 0, p \mapsto 0, q \mapsto \cdots\}$$

Checking Other Properties with Tableaux

A formula A is satisfiable iff a tableau for $A = 1$ contains a complete open branch (and iff every tableau for $A = 1$ contains a complete open branch).

A formula A is valid iff there is a closed a tableau for $A = 0$ (and iff every tableau for $A = 0$ is closed).

Formulas A and B are equivalent iff there is a closed tableau for $(A \leftrightarrow B) = 0$ (and iff every tableau for $(A \leftrightarrow B) = 0$ is closed).

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